MATH 1130 Unit Circle Chart

This is just a brief companion to go with the unit circle chart on the next page. Remember that the coordinates of the point on the unit circle corresponding to angle θ are:

$$(\cos\theta, \sin\theta)$$

For example, the chart tells us that $\cos(\pi/2) = 0$, $\sin(5\pi/3) = -\sqrt{3}/2$, etc.

Also remember that $1/\sqrt{2} = \sqrt{2}/2$, so those can be used completely interchangeably.

Note that these coordinates don't just give you sine and cosine for the labeled angles, but also any other angles that are coterminal with those angles. For example, the angle $420°$ winds up at the same point as $60°$, so $\cos(420°) = 1/2$ and $\sin(420°) = \sqrt{3}/2$.

You don't need to memorize all of the labeled coordinates in this picture, if you are comfortable using reference angles, and deducing $\pm$ signs based on quadrants. For example $150° = 5\pi/6$ makes an angle of $30° = \pi/6$ with the x-axis, so the coordinates of that point are $(\pm\sqrt{3}/2, \pm 1/2)$. However, we know that the point lies in quadrant II, so the x-coordinate is negative and the y-coordinate is positive, hence the coordinates are $(-\sqrt{3}/2, 1/2)$, in other words $\cos(5\pi/6) = -\sqrt{3}/2$ and $\sin(5\pi/6) = 1/2$.

Apart from the four points that lie on the axes, meaning $(1, 0)$, $(0, 1)$, $(-1, 0)$, $(0, -1)$, which don't require any memorization to get the coordinates, all the special points on the unit circle have their x and y coordinates as $\pm 1/2$, $\pm 1/\sqrt{2}$, or $\pm\sqrt{3}/2$. I wrote those in increasing order of size, so I like to think of it as

$$\text{Small: } 1/2, \quad \text{Medium: } 1/\sqrt{2}, \quad \text{Large: } \sqrt{3}/2.$$

So, when I think of obtaining the coordinates of a special point on the unit circle, I just ask, for each coordinate, if it's small/medium/large, and if it's positive or negative based on quadrant. Back to the example of $5\pi/6$, after plotting the point on the circle, I would say "The x-coordinate is large and negative, the y-coordinate is small and positive, so the coordinates are $(-\sqrt{3}/2, 1/2)$." Everyone's brain works differently, but that's how it clicks best for me.

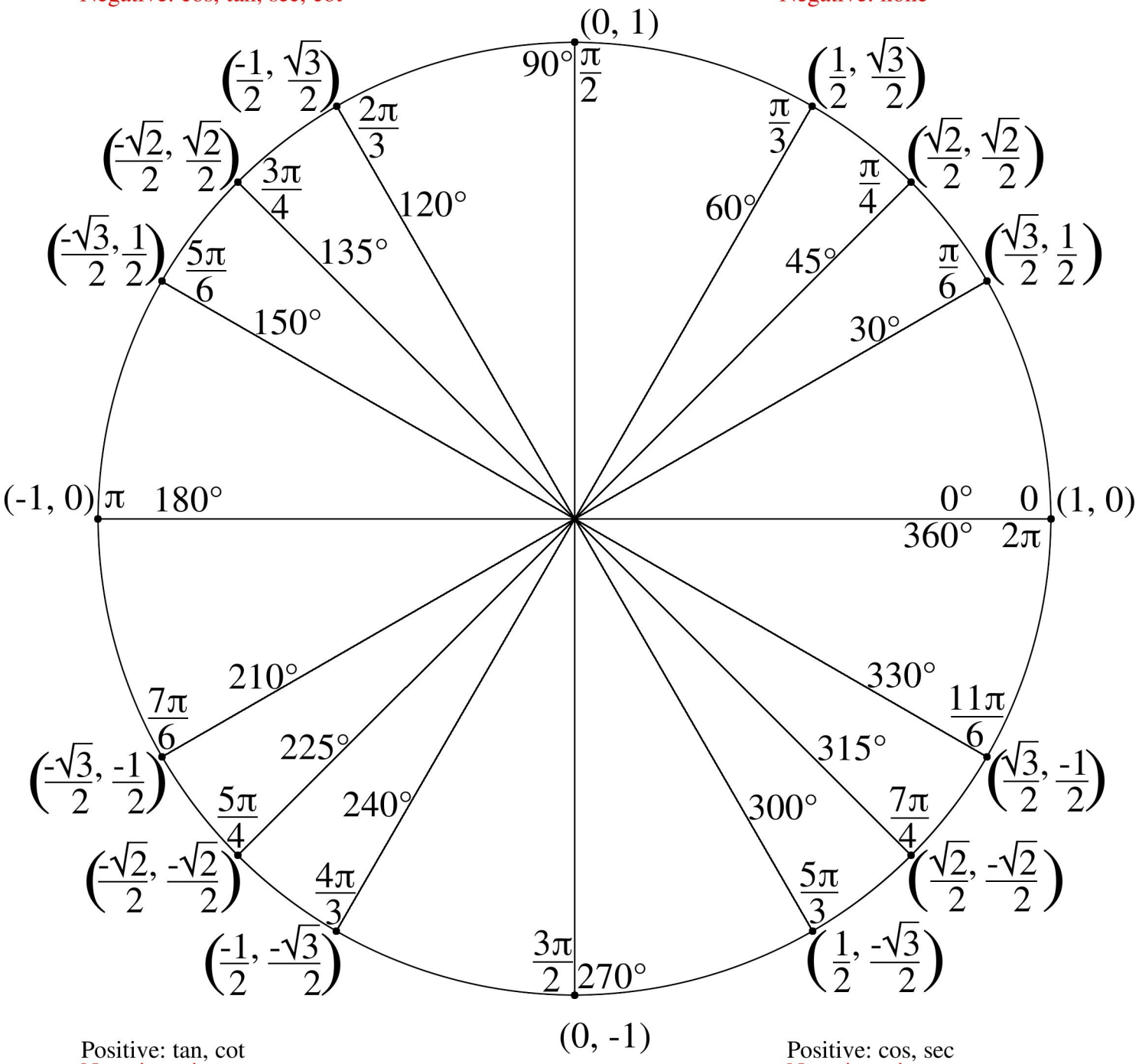
Finally, other than sine and cosine, there are four other trigonometric functions that we have discussed, but they can all be written in terms of sine and cosine. It is important to know these relationships:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}.$$

The Unit Circle

Positive: sin, csc
Negative: cos, tan, sec, cot

Positive: sin, cos, tan, sec, csc, cot
Negative: none



Positive: tan, cot
Negative: sin, cos, sec, csc

Positive: cos, sec
Negative: sin, tan, csc, cot

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