2023 Millsaps College
High School Mathematics Competition

Ciphering Round **Solutions**
10 Problems/3 Minutes Each

- All problems are free response, and 10 points are awarded for each correct answer

- The only things allowed out during the round are the pages from this packet, writing utensils, and scratch paper. In particular, **no calculators or electronic devices of any kind are allowed out during the round.**

- Problems will be worked, and then collected, one at a time. **Do not look at the next page in the packet until directed by a proctor to do so.**

- All work during this round must be done as an individual. **No conversation is allowed during this round.**

Ciphering Round Problems

(1) What is 80% of one-third of 77 more than 2023?

(2) What is the longest distance between two corners of a box with dimensions 7 feet × 24 feet × 60 feet?

(3) On her first three statistics tests of the semester, Ellen had an average score of 86. On her fourth test, she scored a 98. What is her average for all four tests?

(4) Benson buys coffee for his law school study group, and all 13 members of the group ordered either one large latte or one medium latte. Large lattes cost $7 and medium lattes cost $4 (not including sales tax). The sales tax on the order is $5, Benson pays with a $100 bill, and receives $19 in change. How many group members ordered a large latte?

(5) The mean, median, and unique mode of a list of numbers are all different from each other. What is the minimum possible length (meaning the number of numbers) of the list?

(6) Alice and Bob are painting a house together, but Eve wants to stop them. Painting at a constant rate, Alice can paint the house by herself in 4 hours, while Bob can paint the house by himself in 5 hours. Alice and Bob painting together, with Eve simultaneously stripping paint at a constant rate, paint the house in 2.5 hours. How many hours would it take Eve to strip the paint from the entire house?

(7) Assume every day is either cloudy or sunny, not both, and every day is either windy or not windy, not both. If 40% of all days are cloudy, 25% of cloudy days are windy, and 15% of sunny days are windy, what percentage of all days are windy?

(8) Compute $(\sqrt{3} + i)^6$, where $i^2 = -1$. Express your answer in the form $a + bi$, where a and b are real numbers.

(9) Define the function $f(n)$ on positive integers by
$$f(n) = \begin{cases} 3n + 1 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}.$$
What is $f(f(f(f(f(f(f(f(f(f(f(7))))))))))))$?

(10) Find the domain of the function
$$f(x) = \ln(x) + \ln(x^2 - 7x - 18).$$
Express your answer in interval notation.

Ciphering Round Solutions

(1) $(0.8)(1/3)(2023 + 77) = (0.8)(1/3)(2100) = (0.8)(700) = \mathbf{560}$

(2) By the Pythagorean theorem, the diagonal of the 7×24 face has length $\sqrt{7^2 + 24^2} = 25$. Then, the distance between opposite corners is the hypotenuse of a right triangle with legs 25 and 60, which has length $\sqrt{25^2 + 60^2} = \mathbf{65}$ feet.

(3) Ellen's total on her first three tests is $86 \cdot 3 = 258$, so her total on all four tests is $258 + 98 = 356$, so her average is $356/4 = \mathbf{89}$.

(4) The total with tax was $100 - 19 = 81$, so the total without tax was $81 - 5 = 76$. Letting x denote the number of larges and y denote the number of mediums, we now have $x + y = 13$ and $7x + 4y = 76$. Adding -4 times the first equation to the second gives $3x = 24$, so $x = \mathbf{8}$.

(5) In a list with only three numbers, for there to be a unique mode, one number must appear twice. In that case, that number is also the median. Therefore, there must be at least $\mathbf{4}$ numbers on the list, which is certainly possibly with examples like $0, 0, 2, 10$, which has mean 3, median 1, and unique mode 0.

(6) In one hour, Alice paints $1/5$ of the house, while Bob paints $1/4$ of the house. Let x be the proportion of the house Eve strips in one hour, so $\frac{1}{5} + \frac{1}{4} - x = \frac{2}{5}$, which yields $x = 1/20$. In other words, Eve can strip the whole house in $\mathbf{20}$ hours.

(7) Since 40% of days are cloudy, then 60% of days are sunny. Since 25% of cloudy days are windy, $(40\%)(25\%) = 10\%$ of days are cloudy and windy. Similarly, $(60\%)(15\%) = 9\%$ of days are sunny and windy. Therefore, $10\% + 9\% = \mathbf{19}\%$ of all days are windy.

(8) $(\sqrt{3} + i)^2 = 3 + 2i\sqrt{3} - 1 = 2(1 + i\sqrt{3})$, so
$$(\sqrt{3} + i)^3 = 2(1 + i\sqrt{3})(\sqrt{3} + i) = 2(\sqrt{3} + i + 3i - \sqrt{3}) = 8i,$$
and finally $(\sqrt{3} + i)^6 = \left((\sqrt{3} + i)^3\right)^2 = (8i)^2 = \mathbf{-64}$.

(9) There are a total of 11 f's, so we apply the function 11 times to get
$$7 \to 22 \to 11 \to 34 \to 17 \to 52 \to 26 \to 13 \to 40 \to 20 \to 10 \to \mathbf{5}.$$

(10) For $\ln(x)$ to be defined, we need $x > 0$. For $\ln(x^2 - 7x - 18)$ to be defined, we need $x^2 - 7x - 18 = (x - 9)(x + 2) > 0$, which holds when $x < -2$ or $x > 9$. For both to be defined, we need $x > 9$, so the final answer is $(\mathbf{9}, \infty)$.