



2024 Millsaps College High School Mathematics Competition

Ciphering Round **Solutions** 10 Problems/3 Minutes Each

- All problems are free response, and 10 points are awarded for each correct answer
- The only things allowed out during the round are the pages from this packet, writing utensils, and scratch paper. In particular, **no calculators or electronic devices of any kind are allowed out during the round.**
- Problems will be worked, and then collected, one at a time. **Do not look at the next page in the packet until directed by a proctor to do so.**
- All work during this round must be done as an individual. **No conversation is allowed during this round.**

Ciphering Round Problems

- (1) To begin their NBA careers, hall-of-fame players Kobe Bryant, Isiah Thomas, and Michael Jordan wore jersey numbers 8, 11, and 23, respectively. What is the product of these three jersey numbers?

- (2) A fancy restaurant sells tomahawk steaks for \$100 and truffle mashed potatoes for \$18. On a particular night, between these two menu items, the restaurant sells 35 items and earns a total of \$2024 in revenue. How many steaks did they sell?

- (3) A golfer wants to know the height of a tree in her way. Using a rangefinder tool, she finds that the distance from her ball to the base of the tree (along flat ground) is 80 yards, while the distance from her ball to the top of the tree is 82 yards. How tall is the tree, in feet?

- (4) Two stores, Old Navy and GAP, each sell a hoodie for \$50. Sabrina has four coupons, a 20% coupon and a 10% coupon for each store. When processing multiple discounts, Old Navy computes all percentage discounts based on the item's original price, while GAP processes discounts one at a time, computing each discount based on the current (possibly already discounted) price. Assuming she will use both coupons at either store, how much will Sabrina save by buying the hoodie from Old Navy?

- (5) Say a trapezoid is *agreeable* if both of its two parallel base lengths are integers, and its height is the least common multiple of its two parallel base lengths. What is the area of an agreeable trapezoid with parallel base lengths 20 and 24?

- (6) Boise State University running back Ashton Jeanty gained a total of 1376 rushing yards in his first seven games this season. If he has six more games to play, how many rushing yards must he average per remaining game in order to reach 2000 rushing yards for the season?

(7) Roswell High School is playing Blessed Trinity High School in a women's basketball game. Roswell has 10 players on their roster, while Blessed Trinity has 8. Kate and Anna are sisters and play for Roswell, but their other sister, Lily, plays for Blessed Trinity. Each team selects a starting lineup of five players at random. What is the probability that at least one of the three sisters starts the game? Expressed your answer as a reduced fraction.

(8) The Burj Khalifa, a skyscraper in Dubai, is the world's tallest human-made structure, standing 830 meters. An artist would like to construct a scale model of the building with precisely a millionth the original volume. How tall (in meters) should the artist build their model? Express your answer as a decimal.

(9) If

$$(x - 2)^5 = x^5 + ax^4 + bx^3 + cx^2 + dx + e$$

for all real numbers x , what is $81a + 27b + 9c + 3d + e$?

(10) Suppose Shohei Ohtani, who plays for the Dodgers, hits a homerun in 30% of Dodgers games. When he hits a homerun, the Dodgers win 90% of the time. When he does not hit a homerun, the Dodgers win 50% of the time. The Dodgers played a game on September 27 and won. Given only this information, what is the probability that Ohtani hit a homerun on September 27? Express your answer as a reduced fraction.

Ciphering Round Solutions

- (1) $(8)(11)(23) = \mathbf{2024}$
- (2) Let x be the number of steaks and y be the number of orders of mashed potatoes, so $x + y = 35$ and $100x + 18y = 2024$. Subtracting 18 times the first equation from the second yields $82x = 1394$, so $x = \mathbf{17}$.
- (3) We can form a right triangle where one of the legs is 80 yards connecting the ball to the base of the tree, the other leg is the tree (call its length in yards x), and the hypotenuse is 82 yards connecting the ball to the top of the tree. Therefore $x^2 + 80^2 = 82^2$, so $x^2 = 82^2 - 80^2 = 324$, hence $x = 18$ yards, and the final answer is $3(18) = \mathbf{54}$ feet.
- (4) At Old Navy, the 20% coupon and 10% coupon give discounts of \$10 and \$5, respectively, even when used together, so Sabrina pays $50 - 10 - 5 = 35$ dollars for the hoodie. At GAP, the 20% coupon reduces the price to \$40, so the 10% coupon, applied to the new price, yields a \$4 discount. Therefore, Sabrina pays \$36 for the hoodie at GAP, so she saves **\$1** by shopping at Old Navy.
- (5) The least common multiple of 20 and 24 is 120. The area of a trapezoid is the average of its parallel base lengths, multiplied by its height. Therefore, the area in this case is $(22)(120) = \mathbf{2640}$.
- (6) Jeanty needs $2000 - 1376 = 624$ yards in the remaining 6 games, so he must average $624/6 = \mathbf{104}$ yards per game.
- (7) It is easiest to find the probability that *none* of the sisters start, and subtract the result from 1. Roswell has $\binom{10}{5} = 252$ possible starting lineups, $\binom{8}{5} = 56$ of which include neither Kate nor Anna, for a probability of $56/252 = 2/9$. Blessed Trinity has $\binom{8}{5} = 56$ starting lineups, $\binom{7}{5} = 21$ of which do not include Lily, for a probability of $21/56 = 3/8$. Therefore, the probability that no sister starts is $(2/9)(3/8) = 1/12$, so the probability that at least one sister starts is **11/12**.
- (8) Volume is a three-dimensional measure, so scaling a solid by a factor of c multiplies the volume by a factor of c^3 . Setting $c^3 = 1000000$ yields $c = 100$, so the model should be a 1/100 scale version of the original, and hence should be **8.3** meters tall.
- (9) Letting $P(x) = (x - 2)^5$, we see that $p(3) = 1 = 243 + 81a + 27b + 9c + 3d + e$, so $81a + 27b + 9c + 3d + e = \mathbf{-242}$.
- (10) Since we know the Dodgers won, we need to treat Dodger wins as the full sample space, and the wins where Ohtani homers as “successes”. In other words, our answer is $P(\text{Dodgers win and Ohtani homers})/P(\text{Dodgers win})$. The chance Ohtani homers and the Dodgers win is $(0.3)(0.9) = 0.27$, and the chance Ohtani does not homer and the Dodgers win is $(0.7)(0.5) = 0.35$, so the final answer is $0.27/(0.27+0.35) = \mathbf{27/62}$.