2025 Millsaps College
High School Mathematics Competition

Written Test
20 Problems/90 Minutes

- All problems are multiple choice with five answer choices. 10 points are awarded for each correct answer, 0 points are awarded for each incorrect answer, and 2 points are awarded for each blank answer.

- The only things allowed out during the round are the test packet, writing utensils, and scratch paper. In particular, **no calculators or electronic devices of any kind are allowed out during the round.**

- All work during this round must be done as an individual. **No conversation is allowed during this round.**

- Record all your answers on the answer sheet provided at the back of the test packet. **Only the answer sheet should be turned in.** You may keep the remainder of your test packet.

Written Test Problems

(1) Two positive integers x and y add to 2025. Further, x is 25 more than triple y. What is $x - y$?

(A) 975 (B) 1025 (C) 1175 (D) 1225 (E) None of these

(2) Sabrina bought a dress, originally priced at \$46.60, at a 50% discount. The sales tax, computed as a whole number percentage (for example 12%, but not that) of the discounted price, was less than \$1. The total amount Sabrina paid (discounted price plus sales tax), rounded to the nearest penny, was a whole number of dollars. What was the sales tax rate?

(A) 3% (B) 5% (C) 7% (D) 8% (E) None of these

(3) A box has a square base with side length 20 inches. The height of the box is 25 inches. What is the volume of the box, in cubic inches?

(A) 500 (B) 1000 (C) 5000 (D) 10000 (E) None of these

(4) A touring production of Hamilton staged two performances in Tinytown, at a venue that seats 2025 people. Tickets were only available for Tinytown residents, both performances were filled to capacity, and every Tinytown resident saw at least one show. If 450 residents saw both shows, what is the population of Tinytown?

(A) 2025 (B) 3600 (C) 4050 (D) 4500 (E) None of these

(5) What is the mean of the first 25 positive perfect squares?

(A) 169 (B) 196 (C) 221 (D) 243 (E) None of these

(6) Three fair dice are in a bag. One has four faces, numbered 1 thru 4, one has six faces, 1 thru 6, and one has twelve faces, 1 thru 12. Tommy reaches in the bag, grabs one die at random, and rolls it. What is the probability he rolls a 1?

(A) 1/6 (B) 2/13 (C) 3/22 (D) 1/9 (E) None of these

(7) Suppose x and y satisfy the equation $2^y = 5 \cdot 3^x$. Which of the following best describes y as a function of x?

(A) exponential (B) polynomial (C) linear (D) logarithmic (E) y not a function of x

(8) What is the integer closest to $10 \cdot \sqrt[4]{2025}$?

(A) 66 (B) 67 (C) 68 (D) 450 (E) None of these

(9) In baseball, batting average is computed as (number of hits)/(number of at-bats) and rounded to three decimal places. If Mac has had fewer than 20 at-bats this season, and his batting average is 0.455, how many hits does he have this season?

(A) 5 (B) 7 (C) 9 (D) Multiple possibilities (E) None of these

(10) A set of three real numbers $\{a, b, c\}$ has a total of 7 nonempty subsets. For each subset, the average is computed (for example, the average of $\{a\}$ is a, the average of $\{b, c\}$ is $(b + c)/2$), and the resulting list of 7 averages, in increasing order, is as follows: 5, 18, 28, 29, 31, 41, 51. What is $a + b + c$?

(A) 29 (B) 54 (C) 84 (D) 87 (E) None of these

(11) If x is a real number that is close to n^2, where n is an integer, then a good and easily computed approximation for $\sqrt{x}$ can be obtained from the formula $n + \frac{1}{2n}(x - n^2)$. What is the best approximation for $\sqrt{2035}$ produced by this formula?

(A) 811/18 (B) 406/9 (C) 4151/92 (D) 361/8 (E) None of these

(12) During the 1986-87 NBA season, Michael Jordan scored 3041 points (still the most in a season since 1963) in 82 games. His highest single-game point total was 61, and he scored 50 or more points in a game exactly eight times. Using only this information, what is the maximum number of games in which he could have scored a single-digit number of (i.e. strictly fewer than 10) points? (In reality, that happened 0 times.)

(A) 12 (B) 13 (C) 26 (D) 27 (E) None of these

(13) Recall that $\arcsin(x)$ (also denoted $\sin^{-1}(x)$) is the unique angle $-\pi/2 \leq \theta \leq \pi/2$ satisfying $\sin(\theta) = x$. What is the domain of the function $f(x) = \arcsin(3 - 2^x)$?

(A) $[-1, 1]$ (B) $(1, 2)$ (C) $[1, \infty)$ (D) $(-\infty, \infty)$ (E) None of these

(14) A three-term arithmetic progression is a set of the form $\{x, x + d, x + 2d\}$, where x and d are real numbers. What is the number of integer triples (x, y, z) such that $\{x, y, z\}$ is a three-term arithmetic progression with $1 \leq x < y < z \leq 20$?

(A) 45 (B) 90 (C) 95 (D) 190 (E) None of these

(15) A three-term geometric progression is a set of the form $\{x, rx, r^2x\}$, where x and r are real numbers. What is the number of integer pairs (x, y) such that $\{x, 45, y\}$ is a three-term geometric progression with $1 \leq x < 45 < y$?

(A) 6 (B) 7 (C) 9 (D) 10 (E) None of these

(16) A touring production of Hamilton staged three performances in Tinytown, which has a population of 5000, in a venue that seats 2025 people. Tickets were only available for Tinytown residents, all three performances were filled to capacity, and every Tinytown resident saw at least one show. If 600 residents saw both the first and second show, 425 residents saw both the second and third shows, and 250 residents saw both the first and third shows, how many residents saw all three shows?

(A) 0 (B) 75 (C) 125 (D) 200 (E) None of these

(17) Three fair dice are in a bag. One has four faces, numbered 1 thru 4, one has six faces, numbered 1 thru 6, and one has twelve faces, numbered 1 thru 12. Gene reaches in the bag, grabs one die at random, and rolls it. Given that he rolled a 1, what is the probability that he selected the four-sided die?

(A) 1/2 (B) 1/3 (C) 1/4 (D) 1/12 (E) None of these

(18) A trapezoid has parallel bases of positive integer lengths. The height of the trapezoid is double the difference between the two parallel base lengths. The area of the trapezoid is 225. Find the sum of all possible lengths of the longer parallel base.

(A) 113 (B) 152 (C) 177 (D) 194 (E) None of these

(19) Suppose that instead of birthdays, we focus on *birthweeks*, a number between 1 and 52, inclusive. Assume each birthweek is equally likely (not at all true in reality) and ignore the fact that there are not perfectly 52 weeks in a year. What is the least positive integer n such that, in a group of n people, the probability that some pair of people in the group share a birthweek is greater than 1/2?

(A) 9 (B) 12 (C) 16 (D) 27 (E) None of these

(20) Suppose we say that a date is a *Schur date* if the month number and the day of the month add to the last two digits of the year. For example, October 15, 2025 is a Schur date because $10 + 15 = 25$. Let N be the number of Schur dates that occur between January 1, 2000 and December 31, 2999. What is the sum of the digits of N?

(A) 11 (B) 14 (C) 17 (D) 23 (E) None of these

Written Test Solutions

(1) If $x + y = 2025$ and $x = 3y + 25$, then $(3y + 25) + y = 4y + 25 = 2025$, so $y = 500$, $x = 1525$, and $x - y = 1025$, which is **(B)**.

(2) The discounted price is \$23.30, so the sales tax is 70 cents, which is 3% of \$23.30, rounded to the nearest penny, so the answer is **(A)**.

(3) The volume of a box is length times width times height, which in this case, since the base is square, is $20 \cdot 20 \cdot 25 = 10000$ cubic inches, which is **(D)**.

(4) The two shows totaled $2025 + 2025 = 4050$ attendees, but the 450 residents who saw both shows are counted twice, so the true population is $4050 - 450 = 3600$, so **(B)**.

(5) This can be done with brute force, but faster is the formula for the sum of the first n squares, which is $n(n+1)(2n+1)/6$. Dividing by n gives the mean $(n+1)(2n+1)/6$. Plugging in $n = 25$ yields $(26)(51)/6 = 13 \cdot 17 = 221$, which is **(C)**.

(6) The probability of drawing the four-sided die and rolling 1 is $(1/3)(1/4) = 1/12$. For the other die, the respective probabilities are $(1/3)(1/6) = 1/18$, $(1/3)(1/12) = 1/36$. Adding these, the probability of rolling a 1 is $1/12 + 1/18 + 1/36 = 1/6$, so **(A)**.

(7) Taking the natural logarithm of both sides yields $\ln(2^y) = \ln(5 \cdot 3^x)$, which using rules of logarithms becomes $y \ln(2) = \ln(5) + x \ln(3)$, and hence $y = \frac{\ln(3)}{\ln(2)} x + \frac{\ln(5)}{\ln(2)}$ which is a linear function, so the answer is **(C)**.

(8) Since $2025 = 45^2$ (which if we did not know already was revealed in a ciphering problem), $10 \cdot \sqrt[4]{2025} = 10 \cdot \sqrt{45}$, which could either be simplified to $30\sqrt{5}$, requiring a good approximation of $\sqrt{5}$, or written as $\sqrt{4500}$, which must be rounded to the nearest integer. Either approach can work. Using the latter, note that $67^2 = 4489$, while $68^2 = 4624$, indicating 67 is a much better estimate, and the answer is **(B)**.

(9) The fraction $1/11$ has decimal expansion $.090909\ldots$, so $5/11 = 0.454545\ldots$, which rounds to 0.455. No other fraction with denominator 20 has this property, so **(A)**.

(10) Suppose $a < b < c$. The smallest subset average is a, and the largest is c, so $a = 5$, $c = 51$. The second smallest subset average is $(a + b)/2$, so $(5 + b)/2 = 18$, hence $b = 31$, and $a + b + c = 5 + 31 + 51 = 87$, which is **(D)**.

(11) Again using that $2025 = 45^2$, we can use $n = 45$ and get the approximation $\sqrt{2035} \approx 45 + (1/90)(2035 - 2025) = 45 + 1/9 = 406/9$, which is **(B)**.

(12) In the eight 50+ point games, he scored at most $61 \cdot 8 = 488$ points, so he scored at least $3041 - 488 - 2553$ in the remaining 74 games. In these 74 games, he scores at most 9 in the single-digit games and at most 49 in the others, so letting x denote the number of single-digit games, we have $9x + 49(74 - x) \geq 2553$, which rearranges to $40x \leq 1073$, so $x \leq 26.825$. Since x is a whole number, it is at most 26, which is **(C)**.

(13) The range of sine is $[-1, 1]$, so for $\arcsin(3 - 2^x)$ to be defined, we must have $-1 \leq 3 - 2^x \leq 1$, so $-1 \leq 2^x - 3 \leq 1$, which gives $2 \leq 2^x \leq 4$, and hence $1 \leq x \leq 2$. None of the answer choices are $[1, 2]$, so the answer is **(E)**.

(14) For $x, y = x + d$, and $z = x + 2d$ to all be integers, d must be an integer. Since the difference between x and z is $2d$, they must either both be even or both be odd. Further, every pair $x < z$ of odd integers, and every pair $x < z$ of even integers, each determine a unique arithmetic progression by setting $y = (x + z)/2$. Since there are 10 odd integers and 10 even integers between 1 and 20, includive, the number of arithmetic progressions is $2 \cdot \binom{10}{2} = 90$, which is **(B)**.

(15) Letting $x = 45/r$ and $y = 45r$, we need to count choices for $r = a/b$, where $a > b$ are relatively prime positive integers, and both $45b/a$ and $45a/b$ are integers. In particular, both a and b must be divisors of 45. Taking all these conditions into account, the collection of admissible (a, b) pairs is $(45, 1)$, $(15, 1), (9, 1), (9, 5), (5, 1), (5, 3), (3, 1)$, totaling 7 possible values of r, so the answer is **(B)**.

(16) The three shows totaled $3 \cdot 2025 = 6075$ attendees, but the 600 first-second attendees, the 425 second-third attendees, and the 250 first-third attendees have all been double counted. Subtracting $600+425+250 = 1275$ from 6075 yields 4800. However, anyone who saw all three shows has now been added three times, then removed three times, so must be added back in to get the true population. Since the true population is 5000, the number of three-show attendees is 200, which is **(D)**.

(17) As shown in the solution to question 6, the probability of throwing a 1 is $1/6$, while the probabilitiy of choosing the four-sided die and throwing a 1 is $1/12$, so the conditional probability we seek is $(1/12)/(1/6) = 1/2$, which is **(A)**.

(18) Let $x > y$ denote the two parallel base lengths, so the height is $2(x-y)$, and the area is $\frac{1}{2}(x+y) \cdot 2(x-y) = (x+y)(x-y) = 225$. In particular $(x+y)(x-y)$ is a factorization of 225 into a product of two integers. The only further restrictions are that we cannot have $x + y = x - y = 15$, since $y > 0$, and the difference between the integers is $2y$, so they are either both even or both odd, but since 225 is odd, we already knew that. The factorizations $225 = ab$ with $a > b$ positive integers are $(a, b) = (225, 1), (75, 3), (45, 5), (25, 9)$, yielding $(x, y) = (113, 112), (39, 36), (25, 20), (17, 8)$, so the answer is $113 + 39 + 25 + 17 = 194$, which is **(D)**.

(19) With n people, the probability of *no* shared birthweek is

$$\frac{51}{52} \cdot \frac{50}{52} \cdots \frac{52 - n + 1}{52},$$

as each new person must avoid all of the birthweeks that came before them. Careful approximation gives that if $n = 7$, this product is about 0.57, while if $n = 8$, it is about 0.49, so 8 is the first time the chance of a shared birthday exceeds $1/2$. Since 8 was not included as a choice, the answer is **(E)**.

(20) There are several approaches, some much longer than others. The fastest is to note that every non-leap day of the year is a Schur date exactly once per century. For example, November 1 is a Schur date every year of the form XX12. Leap day (February 29) is never a Schur date because $2 + 29 = 31$ is not divisible by 4. Therefore, the number of Schur dates each century is 365, so the number of Schur dates in ten full centuries is 3650, the sum of the digits is 14, and the answer is **(B)**.

Credit Note: Problems 14, 15, and 20 were contributed by 2023 Millsaps graduate Andrew Lott, now at the University of Georgia. A problem similar to Problem 7 appeared in the UGA high school math tournament in 2019.